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General Certificate of Education (A-level)
June 2011

# Mathematics MFP2

## (Specification 6360)

## Further Pure 2

# Final

# *Mark Scheme*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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**Key to mark scheme abbreviations**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| √or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| –*x* EE | deduct *x* marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

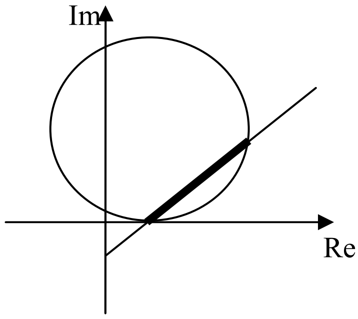
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MFP2**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 1(a) |  | | | Use average of whole question if 2 diagrams used |
| (i) | Circle | B1 | | Circle in any position |
| | correct centre | B1 | | Must be shown |
| | touching $x$-axis | B1F | 3 | ft incorrect centre |
| (ii) | half-line | B1 | | |
| | through (0, –2) | B1 | | Can be inferred |
| | through point of contact of circle with $x$-axis | B1 | 3 | |
| (b) | Inside circle | B1 | | |
| | On line | B1F | 2 | ft errors in position of line and circle |
| | **Total** | | **8** | |
| 2(a) | $\frac{\left(e^{x}+e^{-x}\right)}{2}\frac{\left(e^{y}+e^{-y}\right)}{2}-\frac{\left(e^{x}-e^{-x}\right)}{2}\frac{\left(e^{y}-e^{-y}\right)}{2}$ | M1A1 | | M0 if sinh and cosh confused<br>M1 for formula quoted correctly |
| | Correct expansions | A1 | | Use of $e^{xy}$ A0 |
| | $=\frac{1}{2}\left(e^{x-y}+e^{-(x-y)}\right)=\cosh(x-y)$ | A1 | 4 | AG |
| (b)(i) | $\cosh(x-\ln 2)=\cosh x\cosh(\ln 2)$<br>$-\sinh x\sinh(\ln 2)$ | M1 | | **Alternative:**<br>$\frac{e^{x-\ln 2}+e^{-x+\ln 2}}{2}=\frac{e^{x}-e^{-x}}{2}$ M1 |
| | $\left.\begin{aligned}\cosh(\ln 2)&=\frac{5}{4}\\ \sinh(\ln 2)&=\frac{3}{4}\end{aligned}\right\}$ any method | B1 | | Both correct $e^{x-\ln 2}=\frac{e^{x}}{2}$ or $e^{-x+\ln 2}=2e^{-1}$ used B1 |
| | $\frac{5}{4}\cosh x=\frac{7}{4}\sinh x$ | A1F | | $e^{x}=\sqrt{6}$ A1 |
| | $\tanh x=\frac{5}{7}$ | A1 | 4 | AG $\tanh x=\frac{5}{7}$ A1 |
| (ii) | $x=\frac{1}{2}\ln\left(\frac{1+\frac{5}{7}}{1-\frac{5}{7}}\right)$ or $\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}=\frac{5}{7}$ | M1 | | Could be embedded in (b)(i) |
| | $=\frac{1}{2}\ln 6$ | A1 | 2 | |
| | **Total** | | **10** | |

**MFP2 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **3(a)** | $(r+1)! = (r+1)r(r-1)!$ | M1 | | |
| | Result | A1 | 2 | AG |
| **(b)** | **Attempt** to use method of differences | M1 | | |
| | $\sum_{r=1}^{n}(r^2+r-1)(r-1)! = (n+1)! + n! - 1! - 0!$ | A1 | | |
| | $(n+1)! = (n+1)n!$ | m1 | | Must be seen |
| | $(n+2)n! - 2$ | A1 | 4 | AG |
| | **Total** | | **6** | |
| **4(a)(i)** | $\sum \alpha = 2$ | B1 | | |
| | $\sum \alpha\beta = 0$ | B1 | 2 | |
| **(ii)** | $\sum \alpha^2 = \left(\sum \alpha\right)^2 - 2\sum \alpha\beta$ | M1 | | Used. Watch $\sum \alpha = -2$ (M1A0) |
| | $= 4$ | A1 | 2 | AG |
| **(iii)** | Clear explanation | E1 | 1 | eg $\alpha$ satisfies the cubic equation since it is a root. Accept $z = \alpha$ |
| **(iv)** | $\sum \alpha^3 = 2\sum \alpha^2 - 3k$ | M1 | | Or $\sum \alpha^3 = \left(\sum \alpha\right)^3 - 3\sum \alpha \sum \alpha\beta + 3\alpha\beta\gamma$ |
| | $= 8 - 3k$ | A1 | 2 | AG |
| **(b)(i)** | $\alpha^4 = 2\alpha^3 - k\alpha$ | B1 | | |
| | $\sum \alpha^4 = 2\sum \alpha^3 - k\sum \alpha$ | M1 | | Or $\sum \alpha^4 = \left(\sum \alpha^2\right)^2 - 2\left(\sum \alpha\beta\right)^2 + 4\alpha\beta\gamma\sum \alpha$ |
| | $= 2(8-3k) - 2k$ | A1 | | ft on $\sum \alpha = -2$ |
| | $k = 2$ | A1 | 4 | AG |
| **(ii)** | $\sum \alpha^5 = 2\sum \alpha^4 - k\sum \alpha^2$ | M1 | | |
| | Substitution of values | A1 | | |
| | $= -8$ | A1 | 3 | |
| | **Total** | | **14** | |

**MFP2 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **5(a)** | $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 2x$ | B1 | | Or $\frac{\mathrm{d}y}{\mathrm{d}x} = x\left(x^2+8\right)^{-\frac{1}{2}}$ |
| | $S = 2\pi\int_0^6 y\sqrt{1+\frac{x^2}{y^2}}\ \mathrm{d}x$ | M1<br>A1F | | M1 for use of formula provided $\frac{\mathrm{d}y}{\mathrm{d}x}$ is a function of $x$<br>A1 for substitution for $\frac{\mathrm{d}y}{\mathrm{d}x}$ (one slip) |
| | Eliminating all $y$ | m1 | | |
| | $= 2\sqrt{2}\pi\int_0^6 \sqrt{x^2+4}\ \mathrm{d}x$ | A1 | 5 | AG |
| **(b)** | $\mathrm{d}x = 2\cosh\theta\ \mathrm{d}\theta$ or $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cosh\theta$ | B1 | | |
| | $S = 2\sqrt{2}\,\pi\int\sqrt{4\sinh^2 x+4}\,.\,2\cosh\theta\ \mathrm{d}\theta$ | M1 | | For eliminating $x$ completely and use of $\mathrm{d}\theta$, ie $\mathrm{d}\theta$ attempted |
| | $S = \left(2\sqrt{2}\right)\pi\int 2\cosh\theta.\ 2\cosh\theta\ \mathrm{d}\theta$ | m1 | | Use of $\cosh^2\theta - \sinh^2\theta = 1$<br>(ignore limits) |
| | $= 4\sqrt{2}\,\pi\int\left(\cosh 2\theta + 1\right)\mathrm{d}\theta$ | m1 | | Use of formula for $\cosh 2\theta$; must be correct |
| | $= 4\sqrt{2}\,\pi\left[\frac{\sinh 2\theta}{2}+\theta\right]$ | B1F | | Correct integration of $a\cosh 2\theta + b$ |
| | $= 4\sqrt{2}\,\pi\left[\sinh\theta\cosh\theta + \theta\right]$ | m1 | | Use of $\sinh 2\theta = 2\sinh\theta\cosh\theta$<br>Must be seen |
| | $= 4\sqrt{2}\,\pi\left[\frac{x}{2}\sqrt{\frac{x^2}{4}+1} + \sinh^{-1}\frac{x}{2}\right]_0^6$ | M1 | | Or change limits |
| | $= \pi\left[24\sqrt{5}+4\sqrt{2}\ \sinh^{-1}3\right]$ | A1 | 8 | AG |
| | **Total** | | **13** | |
| **6(a)** | Expansion of $(k+1)\left(4(k+1)^2-1\right)$ | M1 | | Any valid method – first step correct |
| | $= 4k^3+12k^2+11k+3$ | A1 | 2 | AG |
| **(b)** | Assume true for $n=k$<br>For $n=k+1$: | | | |
| | $\sum_{r=1}^{k+1}(2r-1)^2 = \frac{1}{3}k\left(4k^2-1\right)+(2k+1)^2$ | M1A1 | | No LHS M1A0 |
| | $= \frac{1}{3}\left(4k^3+12k^2+11k+3\right)$ | A1F | | ft error in $(2k+1)$ |
| | $= \frac{1}{3}(k+1)\left(4(k+1)^2-1\right)$ | A1 | | Using part (a) |
| | True for $n = 1$ shown<br>Proof by induction set out properly<br>(if factorised by 3 linear factors, allow A1 at this particular point) | B1<br>E1 | 6 | Dependent on all marks correct |
| | **Total** | | **8** | |

**MFP2 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **7(a)(i)** | $\cos 5\theta + \mathrm{i}\sin 5\theta = (\cos\theta + \mathrm{i}\sin\theta)^5$ | M1 | | Attempt to expand 3 correct terms |
| | Expansion in any form | A1 | | Correct simplification |
| | Equate real parts: | m1 | | |
| | $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ | A1 | | AG |
| | Equate imaginary parts: | | | |
| | $\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$ | A1 | 5 | CAO |
| **(ii)** | $\tan 5\theta = \dfrac{\sin 5\theta}{\cos 5\theta}$ | M1 | | Used |
| | Division by $\cos^5\theta$ or by $\cos^4\theta$ | m1 | | |
| | $\tan 5\theta = \dfrac{\tan\theta(5 - 10\tan^2\theta + \tan^4\theta)}{1 - 10\tan^2\theta + 5\tan^4\theta}$ | A1 | 3 | AG |
| **(b)** | $\theta = \dfrac{\pi}{5} \Rightarrow \tan 5\theta = 0$ | M1 | | Or for $\tan^4\theta - 10\tan^2\theta + 5 = 0$ |
| | $\therefore \tan\dfrac{\pi}{5}$ satisfies $t^4 - 10t^2 + 5 = 0$ | A1 | | Or for $\tan 5\theta = 0$ |
| | Other roots $\tan\dfrac{k\pi}{5}$ $k = 2, 3, 4$ | B1 | 3 | OE |
| **(c)** | Product of roots = 5 | M1 | | |
| | $\tan\dfrac{\pi}{5} = -\tan\dfrac{4\pi}{5}$ | B1 | | Or $\tan\dfrac{2\pi}{5} = -\tan\dfrac{3\pi}{5}$ |
| | $\tan^2\dfrac{\pi}{5}\tan^2\dfrac{2\pi}{5} = 5$ | A1 | | |
| | $\tan\dfrac{\pi}{5}\tan\dfrac{2\pi}{5} = +\sqrt{5}$ | A1 | | |
| | **–** sign rejected with reason | E1 | 5 | **Alternative (c)**<br>Use of quadratic formula M1<br>$t^2 = 5 \pm 2\sqrt{5}$ A1<br>$t = \pm\sqrt{5 \pm 2\sqrt{5}}$ B1<br>Correct selection of +ve values E1<br>Multiplied together to get $\sqrt{5}$ A1 |
| | **Total** | | **16** | |
| | **TOTAL** | | **75** | |